One-Dimensional Particulate Electrogasdynamics

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A theoretical analysis of a two-phase particulate electrogasdynamic flow is presented. The effect of the space charge induced electric field force on the solid particle charge carrier is included. The direct conversion of kinetic energy into electricity is accounted for by the electrical body force work term in the energy equation. A sample solution to the seven, ordinary, nonlinear, coupled, differential equations is presented for a typical electrogasdynamic energy conversion channel, $L/D = 13$. Results for the axial pressure, electric current, and potential distribution are given, as well as particulate and gas phase velocities and temperatures for 100 μ diameter particles at two mass-loading ratios of $M_P^* = 0.1$ and $M_P^* = 1.0$.

Nomenclature

A = flow channel cross-sectional area, m^2
B_F = body force nondimensional parameter, dimensionless
Bg = interphase heat-transfer parameter, dimensionless
c_g = specific heat of gas phase at constant pressure, cal/g°K
D_H = flow channel hydraulic diameter, m
D = flow channel diameter $D = D_H$ for a cylindrical flow channel, m
d_p = particle diameter, m
E = electric field strength, V/m
f = friction factor, dimensionless
$\mathbf{F}$ = body force, N/kg
I = electric current, amp
J = electric current density, amp/m^2
k = thermal conductivity of gas phase, cal/cm-sec-°K
L = flow channel length from attractor exit to collector exit, m
l = conversion channel length from attractor exit to collector entrance, m
M_P^* = mass loading ratio, dimensionless
$\mathfrak{M}_i$ = molecular weight of species i
$\mathfrak{M}$ = mean molecular weight of mixture
m_p = mass of a single particle, kg
$\dot{m}_i$ = mass flux of species i, kg/sec
$(N_{Nu})_d$ = Nusselt number based on particle diameter, dimensionless
P = static pressure, N/m^2
q = electric charge per charge carrier, C
R = electrical load resistance, Ω
r = radial coordinate, m
T = absolute temperature, °K
$\hat{\tau}$ = stress tensor, N/m^2
u_{ij} = component species velocity ($i = x,r; j = p,g$) m/sec
$\mathbf{v}_i$ = species velocity, m/sec
$\mathbf{v}$ = mass average velocity, m/sec
$\mathbf{V}_i$ = diffusion velocity, m/sec
w_i = mass fraction of species i, dimensionless
x = axial coordinate, m
ϵ_0 = permittivity of free space, C/V-m
μ = charge carrier mobility, m^2/V-sec
ρ = density of mixture ($\rho = \Sigma_i \rho_i$), kg/m^3
ρ_i = species density ($i = g,p$), kg/m^3
Φ = electrical potential, V

Subscripts

c = referred to the corona
g = gaseous species
L = referred to the load resistance
o = nondimensionalizing, plenum condition
p = particle species
r = radial direction
scr = radial space charge-induced component
scx = axial space charge-induced component
x = axial direction

Superscript

$*$ = nondimensional

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Introduction

Electrogasdynamic energy conversion (EGD) is concerned with the concept of converting the kinetic energy of a unipolar charge-seeded gas stream into electricity. This is accomplished as work is done by the fluid in moving the charges against an electric field. When a molecular charge carrier is used, for example, electrons attached to oxygen molecules in air, the simple one-dimensional diffusion theory[1] shows that the coupling between the charge carriers, oxygen molecules, and the neutral gas can be expressed in terms of a mobility. This implies a no-slip condition between the charge carrier and the gas in the absence of the electric field. In particulate systems, a slip can exist between phases even if the system is electrically neutral. This condition becomes pronounced in accelerating particulate flows which are typical of EGD systems. If the charge carrier is the particulate phase, the coupling between the charge carrier and gas must now be obtained through the simultaneous solution of the particle and gas continuity, momentum, and energy equations. The present work deals with the development of such a set of relations governing the flow parameters of a two-phase particulate EGD fluid. A typical solution is presented.

Governing Relations

The relations for a one-dimensional inviscid particulate flow are considered for the case where an electric field body force acts on the charge which is carried by the particulate phase. The conservation equations for a two-phase, binary system follow directly from the general conservation equations (discussion given in Appendix A). Consistent with previous treatments of particulate flows,[2] the continuum approach is employed with the exception that the solid, particulate species has no associated pressure, i.e.,

$$p = \Sigma_i p_i = p_g \tag{1}$$

The species continuity equations and equation of state are

$$\dot{m}_g = \rho_g u_{xg} A \tag{2}$$

$$\dot{m}_p = \rho_p u_{xp} A \quad (3)$$

and

$$p = \rho_g R T_g \quad (4)$$

The over-all momentum and the single particle momentum equations are, respectively,

$$d(\dot{m}_g u_{xg}/A + \dot{m}_p u_{xp}/A) = -dp + (\rho_p q E_x/m_p)dx + (2f\rho_g u_{xg}^2/D_H)dx \quad (5)$$

and

$$u_{xp} du_{xp}/dx = qE_x/m_p + (C_D \rho_g \pi d_p^2/8m_p)|u_{xg} - u_{xp}|(u_{xg} - u_{xp}) \quad (6)$$

Neglecting heat transfer from the wall to the gas and axial heat conduction, the total energy equation becomes

$$d[\dot{m}_g(c_g T_g + u_{xg}^2/2)/A + \dot{m}_p(c_p T_p + u_{xp}^2/2)/A] = (\rho_p q u_{xp} E_x/m_p)dx \quad (7)$$

and the energy equation for a single particle is

$$u_{xp} dT_p/dx = q u_{xp} E_x/(m_p c_p) + (N_{Nu})_d k\pi d_p(T_g - T_p)/(4m_p c_p) \quad (8)$$

The appropriate Maxwell relations are used to express the current density and electric field, i.e.,

$$\nabla \cdot \mathbf{J} = 0 \quad (9)$$

and

$$\nabla \cdot \mathbf{E} = \rho_p q/(m_p \epsilon_0) \quad (10)$$

In an EGD converter there is a loss of charge to the conversion process via the flow-channel boundary layer. This effect occurs in the radial direction, normal to axial flow for which the one-dimensional gas dynamic theory has been developed, and can be accounted for by combining Eqs. (9) and (10) with the mobility in order to obtain a particle velocity in the radial direction. Incorporating this type of two-dimensional loss in a one-dimensional theory is analogous to the use of a heat-transfer coefficient in a one-dimensional flow. The presence of charge in the flow channel creates electric fields in both the axial and radial directions, the radial contributions resulting solely from space charge, i.e., $E_{SCR} = E_r$. Previous investigations have shown that the gradient of the electric field in the axial direction can be assumed small with respect to the gradient in the radial direction,[3] i.e.,

$$[(\partial E_x/\partial x)/\partial(rE_r)/(r\partial r)] < 1$$

In cylindrical coordinates Eq. (10) then becomes

$$\partial[rE_r(r,x)]/(r\partial r) = \rho_p q/(m_p \epsilon_0)$$

and upon integration yields

$$E_r(r,x) = \rho_p q r/(2m_p \epsilon_0) \quad (11)$$

It should be noted that the particle velocity in the radial direction is considered to be only a result of the radial electric field driving force and in the absence of such a field, $u_{rp} = u_{rg} = 0$. As a consequence the radial particle velocity is related to the electric field through the charge carrier mobility which is assumed to be constant (further discussion of the mobility concept is given in Appendix B), i.e.,

$$u_{rp} = \mu E_r = \mu \rho_p q r/(2m_p \epsilon_0) \quad (12)$$

Since the current density is given in terms of the particle velocity ($\mathbf{J} = \rho_p q \mathbf{u}_p/m_p$) Eq. (9) takes the form

$$\partial(\rho_p q u_{xp}/m_p)/\partial x + \partial(r\rho_p q u_{rp}/m_p)/(r\partial r) = 0 \quad (13)$$

where substitution of Eq. (12) yields

$$dJ_x/dx = -\mu J_x^2/(u_{xp}^2 \epsilon_0)$$

which assumes the following form for a constant area flow channel; i.e.

$$dI_x/dx = -\mu I_x^2/(u_{xp}^2 \epsilon_0 A) \quad (14)$$

Using superposition, the expression for the axial electric field can be given in terms of the potential drop between the attractor and collector and the axial space charge field, i.e.,

$$E_x = \Phi_L/l + E_{scx} \quad (15)$$

Calculation of the current via Eq. (14) eliminates all current flow in the radial direction from usable application in the conversion process. In essence the wall of the conversion channel has been assumed to be part of the attractor, and any charge reaching the wall is lost to ground. Physically, this corresponds to charge forced upstream in the region of the channel wall because of the reduced drag forces in this region. The one-dimensional gas dynamic relations are left unaltered by the addition of the radial charge loss.

Nondimensionalization of Equations

For particulate flow, the plenum chamber (stagnation) conditions are used to nondimensionalize the gas dynamic parameters.[2] In order to maintain consistency, the electrical parameters, current, potential, and field, are nondimensionalized relative to their respective value at the flow channel entrance (Fig. 1). Thus,

$$x^* = x/L, \quad p^* = p/p_o, \quad T_g^* = T_g/T_o, \quad T_p^* = T_p/T_o, \quad u_{xg}^* = u_{xg}/U_o$$

$$u_{xp}^* = u_{xp}/U_o, \quad A^* = AU_o p_o/(\dot{m}_g R T_o), \quad I_x^* = I_x/I_c, \quad E_x^* = E_x/E_x|_{x=0}$$

$$\Phi^* = \Phi/(lE_x|_{x=0}), \quad U_o \equiv [2c_g T_o]^{1/2}, \quad c \equiv c_p/c_g, \quad I_c = I_x|_{x=0}$$

The nondimensional governing relations are as follows: combining the equations of continuity (2) and state (4) yields

$$p^* u_{xg}^* A^* = T_g^* \quad (16)$$

Equation (5) becomes

$$(2fLu_{xg}^*/D)dx^* = -(RA^*/2c)dp^* - du_{xg}^* - M_p^* du_{xp}^* + (B_F M_p^* I_x^* E_x^*/u_{xp}^*)dx^* \quad (17)$$

where

$$B_F = LI_c E_x|_{x=0}/(\dot{m}_p U_o^2) \quad (18)$$

The particle momentum equation (6) is

$$u_{xg}^* u_{xp}^* du_{xp}^* = E_D|u_{xg}^* - u_{xp}^*|(u_{xg}^* - u_{xp}^*)dx^* + B_F u_{xg}^* I_x^* E_x^* dx^* \quad (19)$$

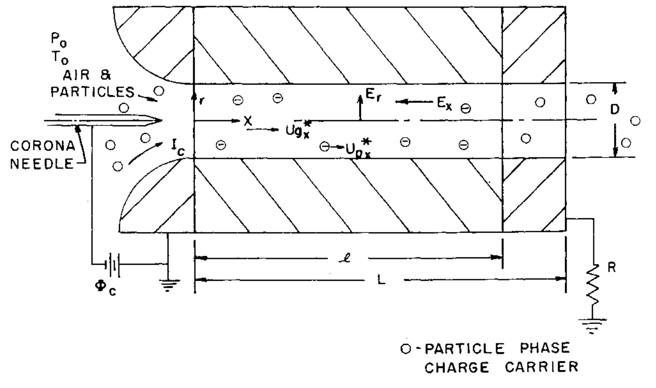

Fig. 1 Electrogasdynamic energy conversion system.

where

$$E_D = C_D \dot{m}_g L d_p^2/(2m_p D^2 U_o) \qquad (20)$$

Equation (7) becomes

$$(p^*A^* + 2u_{xg}^*)du_{xg}^* = -M_p^* c dT_p^* - 2M_p^* u_{xp}^* du_{xp}^* - u_{xg}^* A^* dp^* + 2M_p^* B_F I_x^* E_x^* dx^* - T_g^* dA^*/A^* \qquad (21)$$

Equation (8) becomes

$$cu_{xp}^* dT_p^* = 2B_F u_{xp}^* I_x^* E_x^* dx^* + cB_G(T_g^* - T_p^*)dx^* \qquad (22)$$

where

$$B_G = \pi d_p k L (N_{Nu})_d/(m_p c_p U_o) \qquad (23)$$

Equation (14) becomes

$$dI_x^*/dx = -4\mu L I_c I_x^{*2}/(\epsilon_0 \pi D^2 U_o^2 u_{xp}^{*2}) \qquad (24)$$

The axial space charge electric field can be computed from Gauss' law. A previous consideration yielded the following:[4]

$$E_{scx}(x^*L/l) = I_c L M(x^*L/l)/(2u_{xp} A \epsilon_o) \qquad (25)$$

where

$$M(x^*L/l) = \{(1/g)\ln[(1 + gx^*L/l)^2/(1 + g)] + (2/g)\{\tanh^{-1}\alpha + \tanh^{-1}\gamma + (2/s)(\tanh^{-1}[(G-2)/s] + \tanh^{-1}[(-2\gamma - G)/s]\}\}$$

and $g = Ic/I_L - 1$, $\alpha = \tan[\tan^{-1}(2x/D)/2]$, $G = -gD/(l + gx^*L)$, $\gamma = \tan\{\tan^{-1}[2(l - x)/D]/2\}$, $s = (G^2 + r)^{1/2}$.

In nondimensional form

$$E_{scx}^*(x^*L/l) = E_{scx}/E_x|_{x=0} \qquad (26)$$

and

$$E_x^* = \Phi_L/lE_x|_{x=0} + E_{scx}^* \qquad (27)$$

where $E_x|_{x=0}$ is obtained by evaluating $E_{scx}(x^*L/l)$ at $x^* = 0$ and adding the constant Φ_L/l. The nondimensional potential is

$$d\Phi^* = LE_x^* dx^*/l \qquad (28)$$

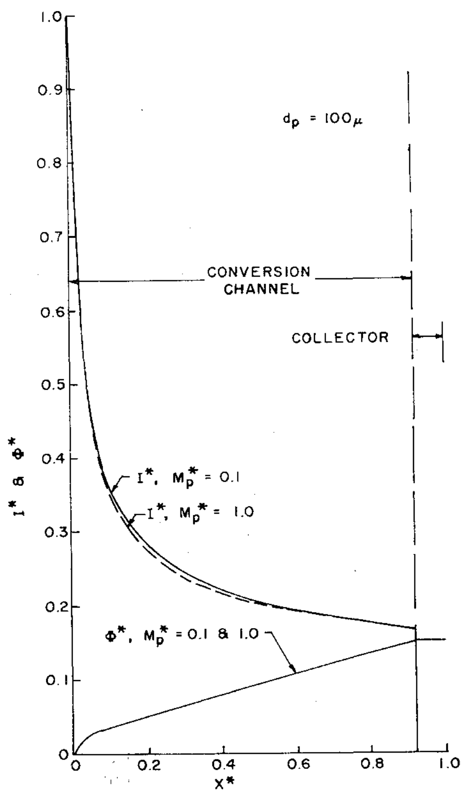

Fig. 2 Axial electric potential and current distribution.

Method of Solution

The aforementioned set of governing Eqs. (16, 17, 19, 21, 22, 24, and 28) represents a set of seven, coupled, ordinary, nonlinear, differential equations in eight unknowns, p^*, u_{xg}^*, u_{xp}^*, T_g^*, T_p^*, f, I^*, and Φ^*. The governing set of equations can be solved numerically; for example, using the Runge-Kutta technique if the axial variations of one of the unknown parameters is known. Previous investigations[2,5] of internal, particulate flows have shown that numerical solutions can be most easily effected in terms of the axial static pressure, i.e., dp^*/dx^* due to its relative ease of experimental determination. It has also been demonstrated[6] that a valid solution of the governing set of gas dynamic relations can be obtained by using average friction factor $\bar{f}$ which was the approach taken here. When the EGD system is considered, the charge carrier mobility must also be specified.

Typical EGD Results

A sample solution to the governing equations was obtained for a constant cross-sectional area flow channel, 6.35 mm in diameter and 76.2 mm long, Fig. 1. The cylindrical flow channel, fed by a converging nozzle and run at the choked flow condition, is indicative of electrogasdynamic energy conversion systems. The results are presented for 100 μ diam, spherical, glass particles at loading ratios of $M_p^* = 0.1$ and 1.0. Input data for the two cases presented are summarized in Table 1.

The ring collector length is 6.35 mm, Fig. 1, and it is assumed that all charge is removed from the flow at $x^* = 0.923$, Fig. 2. Previous investigations[6] using a 6.35-mm-diam duct with $L/D = 288$ revealed an average friction factor for particulate flows of $\bar{f} = 1.8 \times 10^{-3}$. This result was nearly independent of loading ratio, M_p^*, and was determined for a Reynolds number on the order of 10^4. In the present case of $L/D = 13$ at a Reynolds number on the order of 10^5, an average friction factor of $\bar{f} = 1.5 \times 10^{-3}$ was chosen.

The charge carrier mobility and mass flux were chosen to produce a Mach number of unity at the flow duct outlet, $x^* = 1.0$, and a current ratio of $I_L/I_c = 0.1667$ at $x^* = 0.923$. These parameters were interdependent as the body force affected the gas flow parameters. The 100 μ particle size utilized falls in the relatively large region of the range of particle sizes normally employed in EGD converters. The increased volume and mass allows for a lower, more favorable value of mobility, but at the same time results in low particle velocities in the channel.

Solution to the coupled equations proceeds from initial conditions at the conversion channel entrance, downstream to the flow exit at the collector electrode. The Mach number calculated at this position, $x^* = 1.0$, was equal to 1.0 ± 0.5% for the two cases presented. The gas dynamic parameters,

Table 1 Input parameters for sample solutions

	$M_p^* = 0.1$	$M_p^* = 1.0$
$\dot{m}_g$, kg/sec	2.3×10^{-2}	1.62×10^{-2}
$\dot{m}_p$, kg/sec	2.3×10^{-3}	1.62×10^{-2}
I_c, μamp	30.0	30.0
I_x ($x^* = .92$), μamp	5.0	5.0
R, ohm	5.0×10^9	5.0×10^9
p_o, N/m^2	3.44×10^4	3.44×10^4
T_o, °K	294	294
d_p, μ	100	100
μ, M²/V-sec	3.95×10^{-6}	2.23×10^{-6}
$\bar{f}$	1.5×10^{-3}	1.5×10^{-3}

Figs. 3 and 4, for the EGD fluid exhibit similar trends to those encountered in the studies of electrically neutral particulate flows. The flow of gas and particles accelerates the entire length of the flow channel, and the pressure and gas velocity curves, Fig. 3, exhibit sharp decreases and increases respectively in the channel region for $x^* \geq 0.8$ where compressibility effects become important. The particle velocity is seen to be approximately one order of magnitude lower than the gas velocity for most of the channel length, Fig. 3.

The axial potential distribution shown in Fig. 2 is nearly linear. This result is in accord with experimental results obtained in EGD channels employing guarding rings (electrodes having diameters many times the conversion channel diameter).[7]

The increased loading ratio of $M_p^* = 1.0$ from $M_p^* = 0.1$ results in decreased values of pressure, velocity, and temperature of both species. However, the value of mobility is seen to decrease 44% as M_p^* was increased to 1.0. The increased loading ratio for the same total charge input results in less charge per particle. Both solutions presented represent a load voltage of 25 kv at an electric power output of 0.125 w. At the prescribed plenum pressure the input current of $I_c = 30$ μamp can be obtained at a corona voltage of the order of 3 kv. Thus the voltage amplification ratio, Φ_L/Φ_c, is of the order of 8, and the electrical power efficiency is approximately equal to 1.4, i.e., $\Phi_L I_L/\Phi_c I_c \simeq 1.4$.

Appendix A

The governing conservation equations used in the treatment of a one-dimensional particulate EGD flow follow from the general form of the conservation equations for a binary mixture. The general conservation equations have received various degrees of treatment relative to their completeness; for example, a complete final form is given by Williams.[8] A concise derivation of the general relations and a discussion of the terms which were neglected in the previous development follow.

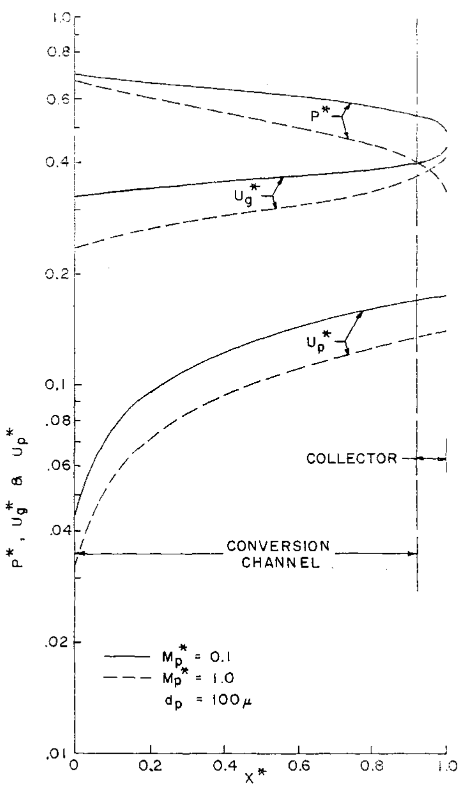

Fig. 3 Species velocity and pressure distribution.

The species velocity ($\mathbf{v}_i$), mass average velocity ($\mathbf{v}$), and the diffusion velocity ($\mathbf{V}_i$), take the form

$$\mathbf{v}_i = u_{xi}\mathbf{i} + u_{yi}\mathbf{j} + u_{zi}\mathbf{k} \tag{A1}$$

$$\mathbf{v} = \Sigma(\rho_i \mathbf{v}_i/\rho) = \Sigma w_i \mathbf{v}_i \tag{A2}$$

$$\mathbf{V}_i = \mathbf{v}_i - \mathbf{v} \tag{A3}$$

The species continuity equation is

$$\partial\rho_i/\partial t + \text{div}(\rho_i \mathbf{v}_i) = r_i \tag{A4}$$

and summing over-all species yields the global continuity equation

$$\partial\rho/\partial t + \text{div}(\rho\mathbf{v}) = 0 \tag{A5}$$

For one-dimensional steady state and in the absence of chemical reaction, Eq. (A4) reduces to Eqs. (2) and (3). The conservation of momentum written for each species and summed over all species is

$$\sum_i [\partial(\rho_i \mathbf{v}_i)/\partial t + \text{div}(\rho_i \mathbf{v}_i \mathbf{v}_i) - \rho_i \mathbf{F}_i] = -\nabla p - \text{div}\hat{\tau} \tag{A6}$$

where

$$p = \sum_i p_i$$

and the stress tensor is defined as

$$\hat{\tau} = \sum_i \hat{\tau}_i$$

Replacing the stress tensor by a friction factor and assuming the body force, i.e., the electric field force $F_{px} = qE_x/m_p$, only acts on the particulate phase, Eq. (A6) reduces to Eq. (5). The total energy equation for the mixture is given by

$$\sum_i [\partial\rho_i(h_i + \mathbf{v}_i^2/2)/\partial t + \text{div}\rho_i\mathbf{v}_i(h_i + \mathbf{v}_i^2/2) + \text{div}\hat{\tau}_i\mathbf{v}_i - \rho_i\mathbf{F}_i\cdot\mathbf{v}_i] = \partial p/\partial t - \text{div}\mathbf{q} \tag{A7}$$

where the heat flux vector $\mathbf{q}$ is given by $\mathbf{q} = \sum_i \mathbf{q}_i$ and $\mathbf{v}_i^2 \equiv \mathbf{v}_i\cdot\mathbf{v}_i = |\mathbf{v}_i|^2$

Neglecting shear work and heat conduction, Eq. (A7) reduces to Eq. (7) where the body force work has been included. In an

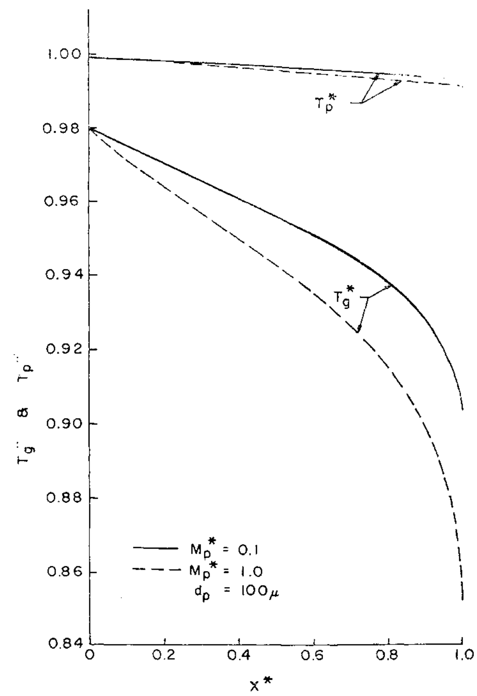

Fig. 4 Species temperature distribution.

EGD flow this term $(\rho_p q u_{xp} E_x/m_p)$ accounts for the conversion of the fluid kinetic energy to electrical energy. When dealing with two phase particulate flow systems, the species velocities are of most interest and Eqs. (A4–A7) may be used without further manipulation. However, the equations can be cast into a more familiar form by eliminating the species velocities in favor of the mass average velocity. The resulting equations are of the form of the conservation equations for a single species fluid, plus a series of diffusion terms, where the mass average velocity has replaced the fluid velocity.

The following substantial derivatives are defined

$$D/Dt \equiv \partial/\partial t + \mathbf{v}\cdot\nabla$$

and

$$D/Dt_i \equiv \partial/\partial t + \mathbf{v}_i\cdot\nabla$$

Now, for any function K the "fundamental identity"[9] is

$$\partial\rho_i K/\partial t + \mathrm{div}(\rho_i \mathbf{v}_i K) = \rho_i(DK/Dt_i) + r_i K \quad \text{(A8)}$$

where r_i is the rate of production of species i. Applying relation (A8) to conservation equations (A6) and (A7) yields

$$\sum_i \{\rho_i(D\mathbf{v}_i/Dt_i) - \rho_i\mathbf{F}_i + r_i\mathbf{v}_i\} = -\nabla p - \mathrm{div}\hat{\tau} \quad \text{(A9)}$$

and

$$\sum_i \{\rho_i(D/Dt_i)(h_i + \mathbf{v}_i^2/2) + \mathrm{div}\hat{\tau}_i\mathbf{v}_i - \rho_i\mathbf{F}_i\cdot\mathbf{v}_i + r_i(h_i + \mathbf{v}_i^2/2)\} = \partial p/\partial t - \mathrm{div}\mathbf{q} \quad \text{(A10)}$$

The use of fundamental identity to obtain an expression similar to Eq. (A10) has also been considered by Soo.[2] The fundamental identity for any function $K = \sum_i m_i K_i$ is given by

$$\rho DK/Dt = \sum_i [\rho_i(DK_i/Dt_i) + r_i K_i - \mathrm{div}(\rho_i \mathbf{V}_i K_i)] \quad \text{(A11)}$$

The kinetic energy based on mass average velocity may be written in terms of the kinetic energy of diffusion, KE^D,

$$(\mathbf{v}\cdot\mathbf{v})/2 = \sum_i w_i[(\mathbf{v}_i\cdot\mathbf{v}_i/2) - KE_i^D] \quad \text{(A12)}$$

where

$$KE_i^D = (\mathbf{V}_i\cdot\mathbf{V}_i)/2$$

and

$$KE^D = \sum_i w_i KE_i^D$$

Also, the diffusion stress tensor is defined as

$$\hat{\tau}_i^D \equiv \rho_i\mathbf{V}_i\mathbf{V}_i$$

and

$$\hat{\tau}^D = \sum_i \hat{\tau}_i^D$$

In view of relation (A12) the energy Eq. (A10) may be rewritten

$$\sum_i \{\rho_i(D/Dt_i)(h_i + \mathbf{v}_i^2/2 - KE_i^D) + \rho_i(D/Dt_i)KE_i^D + \mathrm{div}\hat{\tau}_i\mathbf{v}_i - \rho_i\mathbf{F}_i\cdot\mathbf{v}_i + r_i(h_i + v_i^2/2)\} = \partial p/\partial t - \mathrm{div}\mathbf{q} \quad \text{(A13)}$$

Using the fundamental identity, Eq. (A11), in Eq. (A9) and twice in Eq. (A13) yields the following result for the momentum and energy equations, respectively,

$$\rho(D\mathbf{v}/Dt) = -\nabla p - \mathrm{div}(\hat{\tau} + \hat{\tau}^D) + \sum_i \rho_i\mathbf{F}_i \quad \text{(A14)}$$

$$\rho(D/Dt)[h + \mathbf{v}^2/2 + KE^D] = \partial p/\partial t + \sum_i\{\mathrm{div}(-\hat{\tau}_i\mathbf{v}_i) + \rho_i\mathbf{F}_i\cdot\mathbf{v}_i - \mathrm{div}\rho_i\mathbf{V}_i(h_i + \mathbf{v}_i^2/2)\} - \mathrm{div}\mathbf{q} \quad \text{(A15)}$$

The following identities prove useful in further reducing the form of the energy equation (A15)

$$\sum_i \rho_i\mathbf{V}_i(\mathbf{v}_i^2/2) = \sum_i \rho_i\mathbf{V}_i(\mathbf{V}_i^2/2) + \hat{\tau}^D\mathbf{v} \quad \text{(A16)}$$

and

$$\sum_i \mathrm{div}\hat{\tau}_i\mathbf{v}_i = \sum_i \mathrm{div}\hat{\tau}_i\mathbf{V}_i + \mathrm{div}\hat{\tau}\mathbf{v} \quad \text{(A17)}$$

Employing relations (A16) and (A17) in Eq. (A15), the final form of the total energy equation is obtained.

$$\rho(D/Dt)(h + \mathbf{v}^2/2 + KE^D) = \partial p/\partial t - \mathrm{div}[(\hat{\tau} + \hat{\tau}^D)\mathbf{v}] + \mathbf{v}\cdot\sum_i\rho_i\mathbf{F}_i - \mathrm{div}\mathbf{q} + \sum_i[\rho_i\mathbf{V}_i\cdot\mathbf{F}_i - \mathrm{div}\hat{\tau}_i\mathbf{V}_i - \mathrm{div}\rho_i\mathbf{V}_i KE_i^D] \quad \text{(A18)}$$

where the term $\Sigma\rho_i\mathbf{V}_i h_i$ has been incorporated into the heat flux vector $\mathbf{q}$. Equations (A4, A5, A14, and A18) represent the conservation equations in the most used form.

Using the species form of the energy Eq. (A7) in lieu of relation (A18) effectively accounts for the kinetic energy of diffusion terms usually omitted from Eq. (A18) on an order of magnitude argument. Equations (A14) and (A18) have been included in the preceding analysis to add completeness and to exhibit this concise derivation.

Appendix B

If the difference in species velocities is only a result of diffusion, then considering only the effect of body force on diffusion, the expression for mass flux becomes

$$\rho_p\mathbf{V}_g = -\rho^2 w_g w_p \mathfrak{M}_g\mathfrak{M}_p D_{pg}\mathbf{F}_p/p\mathfrak{M}^2 \quad \text{(B1)}$$

where D_{pg} is the binary diffusion coefficient. Substituting from Eq. (A3) for $\mathbf{V}_g$, and using Eq. (A2) for $\mathbf{v}$, after rearrangement Eq. (B1) becomes

$$\mathbf{v}_g = \mathbf{v}_p - \rho\mathfrak{M}_g\mathfrak{M}_p D_{pg}\mathbf{F}_p/p\mathfrak{M}^2 \quad \text{(B2)}$$

The difference between the gas and particle velocities, i.e., the charge carrier lag, can be expressed in terms of a mobility and the electric field force which is the body force $\mathbf{F}_p$ acting on the particulate charge carrier phase. Thus Eq. (B2) becomes

$$\mathbf{v}_p = \mathbf{v}_g + \mu\mathbf{E} \quad \text{(B3)}$$

where the mobility μ is given by

$$\mu = n\mathfrak{M}_g\mathfrak{M}_p D_{pg}/p\mathfrak{M}^2 w_p \quad \text{(B4)}$$

and

$$n = \rho_p q/m_p \quad \text{(B5)}$$

In an accelerating EGD particulate flow the criterion necessary for the implementation of Eq. (B2) is satisfied in the radial direction only.

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Langmuir Probe Response in a Turbulent Plasma

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A theoretical and experimental investigation of the response of collisionless cylindrical Langmuir probes in unsteady plasmas is made with the specific aim of developing methods for measuring the mean and statistical plasma properties. A comprehensive theory describing the transient responses of both single-probes and symmetric double-probes to arbitrary (low-frequency) fluctuations in electron density, plasma potential, electron temperature, and ion temperature is formulated by perturbing the probe steady-state equations about the mean plasma properties. The ratios of probe radius to debye length and applied potential to electron temperature are used as controllable parameters to form a set of equations from which the mean and rms properties may be determined. Experiments are performed in an unsteady highly expanded low-density flowing argon plasma. Pertinent features of the theory are verified and quantitative measurements of the mean and fluctuating plasma properties are made.

Nomenclature

A = probe area
E_N, E_Θ = normalized partial derivatives appearing in the equations defining the floating potential measurement
F_N, F_Θ = normalized partial derivatives appearing in the equations defining the single-probe current
G_N, G_Θ = normalized partial derivatives appearing in the equations defining the double-probe current
H, h = current in double-probe circuit
I = retarding field electron current
J, j = attractive field probe current (ion or electron)
K = defined by Eq. (5)
L = probe length
M = mass of species
N, n = charged particle density
q = one electronic charge
r = probe radius
t = time
U = freestream velocity
V = applied probe to probe or probe to reference potential
α = defined by Eq. (3)
β = defined by Eq. (4)
Θ, θ = temperature (eV)
λ = $[\Theta/(4\pi Nq)]^{1/2}$, electron debye length
τ = defined by Eq. (8)
Φ, ϕ = potential measured with respect to the plasma potential
χ = Φ/Θ nondimensionalized potential
ψ = V/Θ nondimensionalized potential

Subscripts

$+, -$ = positive ion, electron
1,2 = numerical designation for double-probes
f = open circuit potential
m = measured quantity
o = computed using mean plasma properties
p = plasma
w = probe surface

Superscripts

$'$ = ion current to a probe that is common to a double-probe system
$-$ = denotes time average

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1. Introduction

FLUCTUATIONS or instabilities are an important characteristic of many interesting plasmas. Because the free-molecule Langmuir probe has proved successful in measuring the local properties of steady plasmas, attempts have been made to use these probes to measure the mean and statistical properties of unsteady plasmas. Considerable work in this direction has been done where the earlier authors[1–4] attempted to predict the average probe current for fluctuations in electron density, plasma potential, and electron temperature. These analyses were based on the steady-state theories of Bohm[5] and of Langmuir and Mott-Smith,[6] thus restricting the attractive-field probe current collection (ion or electron) to the "thin sheath limit" (TSL) or the "orbital motion limit" (OML). Demetriades and Doughman[7] gave attention to the problem of measuring the actual magnitudes of the fluctuations. Beginning with the steady-state theory of Langmuir and Mott-Smith,[6] they found the influence of fluc-